

Question 3

Let

$$f(x) = \frac{e^x}{\cos x} + \ln(x^2 + 1).$$

- (a) Find $f'(x)$.
- (b) Determine the domain of $f'(x)$.

Question 3 - Solution

We are given:

$$f(x) = \frac{e^x}{\cos x} + \ln(x^2 + 1)$$

(a) Find $f'(x)$:

Differentiate each term separately.

For the first term $\frac{e^x}{\cos x}$, use the quotient rule:

$$\frac{d}{dx} \left(\frac{e^x}{\cos x} \right) = \frac{(\cos x)(e^x) - e^x(-\sin x)}{(\cos x)^2} = \frac{e^x(\cos x + \sin x)}{\cos^2 x}$$

For the second term $\ln(x^2 + 1)$, use the chain rule:

$$\frac{d}{dx} \ln(x^2 + 1) = \frac{2x}{x^2 + 1}$$

Combine both:

$$f'(x) = \frac{e^x(\cos x + \sin x)}{\cos^2 x} + \frac{2x}{x^2 + 1}$$

(b) Domain of $f'(x)$:

We must exclude values that cause division by zero.

$\cos x = 0$ when $x = \frac{\pi}{2} + n\pi$, where $n \in \mathbb{Z}$, $x^2 + 1 > 0$ for all real x , so no restrictions there

Domain: All real x such that $\cos x \neq 0$

$$f'(x) = \frac{e^x(\cos x + \sin x)}{\cos^2 x} + \frac{2x}{x^2 + 1}, \quad x \in \mathbb{R} \setminus \left\{ \frac{\pi}{2} + n\pi \mid n \in \mathbb{Z} \right\}$$