

Question 2

Let

$$f(x) = \sqrt{x^2 + 3x} + \frac{1}{\sqrt{x^2 + 3x}}.$$

- (a) Compute $f'(x)$ using differentiation rules.
- (b) Determine all points where the derivative does not exist.

Question 2 - Solution

Domain. Set $u = x^2 + 3x = x(x + 3)$. The reciprocal square root requires $u > 0$, so

$$D = (-\infty, -3) \cup (0, \infty).$$

Differentiate.

$$f = u^{1/2} + u^{-1/2}, \quad \boxed{f'(x) = \frac{(2x+3)(x^2+3x-1)}{2(x^2+3x)^{3/2}}}, \quad x \in D.$$

The derivative exists at every point of the function's domain. On $[-3, 0]$, the function itself is undefined; these are not nondifferentiable points of its graph.