

Question 1

Let

$$f(x) = \frac{\sin(x)}{x^2 + 1} + x \cos(x).$$

- (a) Find the derivative $f'(x)$ using standard differentiation rules.
- (b) Determine the slope of the tangent line to the graph of $f(x)$ at $x = 0$.

Question 1 - Solution

We are given:

$$f(x) = \frac{\sin(x)}{x^2 + 1} + x \cos(x)$$

(a) Find $f'(x)$:

Differentiate term by term.

First term (quotient rule):

$$\frac{d}{dx} \left(\frac{\sin(x)}{x^2 + 1} \right) = \frac{(x^2 + 1) \cos(x) - \sin(x)(2x)}{(x^2 + 1)^2}$$

Second term (product rule):

$$\frac{d}{dx} (x \cos(x)) = \cos(x) - x \sin(x)$$

Putting both together:

$$f'(x) = \frac{(x^2 + 1) \cos(x) - 2x \sin(x)}{(x^2 + 1)^2} + \cos(x) - x \sin(x)$$

(b) Find the slope at $x = 0$:

Evaluate $f'(0)$.

$$\begin{aligned} f'(0) &= \frac{(0^2 + 1) \cos(0) - 2(0) \sin(0)}{(0^2 + 1)^2} + \cos(0) - 0 \cdot \sin(0) \\ &= \frac{1 \cdot 1 - 0}{1^2} + 1 - 0 \\ &= 1 + 1 = \boxed{2} \end{aligned}$$

So, the slope of the tangent line to the curve at $x = 0$ is $\boxed{2}$.