

Question 10

Suppose that the function $C(p)$ models the total cost (in dollars) of producing p items, and is given by:

$$C(p) = 200 + 8p + 0.05p^2$$

- (a) Find and interpret $C'(p)$.
- (b) Find $C'(50)$ and interpret its meaning in context.
- (c) Determine whether the marginal cost is increasing or decreasing as p increases.

Question 10 - Solution

We are given:

$$C(p) = 200 + 8p + 0.05p^2$$

(a) Find and interpret $C'(p)$:

Differentiate:

$$C'(p) = \frac{d}{dp}(200 + 8p + 0.05p^2) = 0 + 8 + 0.1p = 8 + 0.1p$$

Interpretation: $C'(p)$ is the marginal cost — the approximate cost of producing one additional item when p items have already been produced.

(b) Compute and interpret $C'(50)$:

$$C'(50) = 8 + 0.1(50) = 8 + 5 = \boxed{13}$$

Interpretation: When 50 items have been produced, the cost of producing one more item is approximately \$13.

(c) Is marginal cost increasing or decreasing?

Since:

$$C'(p) = 8 + 0.1p$$

and the derivative of this is:

$$C''(p) = 0.1 > 0$$

The marginal cost is increasing as p increases.

Marginal cost increases with production level.