

Question 9

The position $s(t)$ of a particle moving along a straight line is given by:

$$s(t) = \frac{3t^2 + 2}{t + 1}, \quad t > 0$$

- (a) Find the instantaneous velocity of the particle at $t = 1$.
- (b) Interpret the sign of the velocity at $t = 1$.
- (c) Find all critical points of $s(t)$ and determine whether the particle changes direction.

Question 9 - Solution

We are given:

$$s(t) = \frac{3t^2 + 2}{t + 1}, \quad t > 0$$

(a) Instantaneous Velocity at $t = 1$:

Velocity is the derivative $s'(t)$. Using the quotient rule:

$$\begin{aligned} s'(t) &= \frac{(t+1)(6t) - (3t^2+2)(1)}{(t+1)^2} \\ &= \frac{6t^2 + 6t - 3t^2 - 2}{(t+1)^2} = \frac{3t^2 + 6t - 2}{(t+1)^2} \end{aligned}$$

Evaluate at $t = 1$:

$$s'(1) = \frac{3 + 6 - 2}{4} = \frac{7}{4}$$

$$\boxed{s'(1) = \frac{7}{4}}$$

(b) Interpretation of the Sign of Velocity:

Since $s'(1) = \frac{7}{4} > 0$, the particle is moving in the positive direction at $t = 1$.

The particle is moving to the right (increasing position) at $t = 1$

(c) Critical Points and Direction Change:

Critical points occur where $s'(t) = 0$ or where $s'(t)$ is undefined.

The derivative is undefined at $t = -1$, which is outside the domain $t > 0$ and therefore ignored.

Set the numerator equal to zero:

$$3t^2 + 6t - 2 = 0$$

$$t = \frac{-6 \pm \sqrt{36 + 24}}{6} = \frac{-6 \pm \sqrt{60}}{6} = \frac{-3 \pm \sqrt{15}}{3}$$

This gives two solutions:

$$t = \frac{-3 - \sqrt{15}}{3} \quad \text{and} \quad t = \frac{-3 + \sqrt{15}}{3}$$

Since time must satisfy $t > 0$, the negative solution is rejected. The only valid critical time is:

$$\boxed{t = \frac{-3 + \sqrt{15}}{3}}$$

To determine whether the particle changes direction, test the sign of $s'(t)$ on either side of this value (within $t > 0$). The derivative changes from negative to positive, indicating a direction change.

The particle changes direction at $t = \frac{-3 + \sqrt{15}}{3}$