

Question 8

The height of a ball thrown vertically upward is modeled by the function:

$$h(t) = -16t^2 + 64t + 80$$

where $h(t)$ is the height in feet after t seconds.

- (a) Determine the velocity function of the ball.
- (b) When does the ball reach its maximum height?
- (c) What is the maximum height the ball reaches?
- (d) What is the acceleration of the ball at any time t ?

Question 8 - Solution

We are given:

$$h(t) = -16t^2 + 64t + 80$$

(a) Velocity Function:

The velocity is the first derivative of height:

$$v(t) = h'(t) = -32t + 64$$

$$v(t) = -32t + 64$$

(b) Time of Maximum Height:

The ball reaches its maximum height when the velocity is zero:

$$-32t + 64 = 0 \Rightarrow t = 2$$

$$t = 2 \text{ seconds}$$

(c) Maximum Height:

Plug $t = 2$ into the original height function:

$$h(2) = -16(2)^2 + 64(2) + 80 = -64 + 128 + 80 = 144$$

$$144 \text{ feet}$$

(d) Acceleration Function:

Acceleration is the derivative of velocity:

$$a(t) = v'(t) = -32$$

$$a(t) = -32 \text{ ft/s}^2 \text{ (constant acceleration due to gravity)}$$