

Question 7

The position of a particle moving along a straight line is given by the function

$$s(t) = t^3 - 6t^2 + 9t$$

where $s(t)$ is in meters and $t \geq 0$ is in seconds.

- (a) Find the velocity function $v(t)$.
- (b) At what time(s) is the particle at rest?
- (c) Determine the direction of motion (moving forward or backward) on the intervals determined by part (b).
- (d) Is the particle speeding up or slowing down at $t = 1$? Justify using acceleration.

Question 7 - Solution

Velocity and rest times.

$$v(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3), \quad a(t) = 6t - 12.$$

Hence the particle is at rest at $t = 1, 3$.

For $t \geq 0$, it moves forward on $[0, 1) \cup (3, \infty)$ and backward on $(1, 3)$.

Behavior at $t = 1$. Here $v(1) = 0$ and $a(1) = -6$. Just before 1, velocity and acceleration have opposite signs, so speed decreases. Just after 1, both are negative, so speed increases.

The speed $|v(t)|$ has left derivative -6 and right derivative 6 at 1; it is not differentiable there.

At $t = 1$ it is at rest and reverses direction.

It slows down immediately before that instant and speeds up immediately afterward.