

Question 6

The temperature T (in $^{\circ}\text{C}$) in a chemical reaction chamber is modeled by the function

$$T(t) = 100 - 20e^{-0.5t}$$

where t is the time in minutes since the reaction started.

- (a) Find the rate at which the temperature is changing with respect to time.
- (b) What is the rate of temperature change at $t = 0$?
- (c) What is the long-term behavior of the rate of temperature change? Explain.

Question 6 - Solution

We are given:

$$T(t) = 100 - 20e^{-0.5t}$$

(a) Rate of Change of Temperature:

Differentiate $T(t)$ with respect to t :

$$T'(t) = \frac{d}{dt}(100 - 20e^{-0.5t}) = -20 \cdot \frac{d}{dt}(e^{-0.5t}) = -20 \cdot (-0.5e^{-0.5t}) = 10e^{-0.5t}$$

$$T'(t) = 10e^{-0.5t}$$

This represents the rate of temperature change at time t .

(b) Rate of Change at $t = 0$:

$$T'(0) = 10e^{-0.5 \cdot 0} = 10e^0 = 10$$

$$T'(0) = 10 \quad (\text{°C per minute})$$

Interpretation: At the beginning of the reaction, the temperature is increasing at 10°C per minute.

(c) Long-Term Behavior:

As $t \rightarrow \infty$, $e^{-0.5t} \rightarrow 0$, so:

$$T'(t) = 10e^{-0.5t} \rightarrow 0$$

Conclusion: The rate of temperature change decreases over time and approaches zero. This means the temperature stabilizes and stops changing as time progresses.

$$\lim_{t \rightarrow \infty} T'(t) = 0$$