

Question 5

The population of a certain city (in thousands) $P(t)$, as a function of time t (in years since 2000), is modeled by the function:

$$P(t) = 500 + 40t - 0.5t^2$$

- (a) Find the rate of change of the population with respect to time.
- (b) What is the population growth rate in the year 2010?
- (c) At what year is the population growth rate zero?
- (d) Interpret your result from (c) in the context of the population.

Question 5 - Solution

We are given:

$$P(t) = 500 + 40t - 0.5t^2$$

(a) Rate of Change of Population:

The rate of change of the population is given by the derivative:

$$P'(t) = \frac{d}{dt}(500 + 40t - 0.5t^2) = 40 - t$$

$$P'(t) = 40 - t$$

(b) Growth Rate in 2010:

Since t is years since 2000, 2010 corresponds to $t = 10$.

$$P'(10) = 40 - 10 = 30$$

Interpretation: In 2010, the population was increasing at a rate of 30 thousand people per year.

$$P'(10) = 30 \quad (\text{thousand people/year})$$

(c) When is the Population Growth Rate Zero?

Set $P'(t) = 0$:

$$40 - t = 0 \Rightarrow t = 40$$

$$t = 40$$

(d) Interpretation:

Since $t = 40$ corresponds to the year $2000 + 40 = 2040$, the population stops growing in 2040.

Conclusion: In 2040, the population reaches its maximum and stops increasing. After that, it begins to decline.

$$\text{The population stops growing in 2040.}$$