

Question 4

The cost (in dollars) to produce x items is given by the function:

$$C(x) = 50x + 300\sqrt{x}$$

- (a) Find the marginal cost function.
- (b) Interpret the meaning of $C'(100)$.
- (c) Approximate the cost of producing the 101st item using the derivative.
- (d) Compute the actual cost of producing the 101st item and compare with your approximation.

Question 4 - Solution

Marginal cost. For $x > 0$,

$$C'(x) = 50 + 150/\sqrt{x}.$$

Thus $C'(100) = 65$ dollars per item. This is the instantaneous cost rate and predicts a cost of approximately $\boxed{\$65}$ for one additional item.

Actual additional cost.

$$C(101) - C(100) = 50 + 300(\sqrt{101} - 10) \approx \boxed{\$64.96}.$$

The derivative estimate exceeds the actual cost by about \$0.04. This agrees with $C''(x) = -75x^{-3/2} < 0$: the marginal cost decreases over the next item.