

Question 3

The position of a particle moving along a straight line is given by:

$$s(t) = 3t^3 - 15t^2 + 18t \quad (\text{in meters}), \text{ for } t \geq 0.$$

- (a) Find the velocity and acceleration functions of the particle.
- (b) At what times is the particle at rest?
- (c) Determine the intervals when the particle is moving forward.
- (d) When is the acceleration zero, and what does it signify?

Question 3 - Solution

Given:

$$s(t) = 3t^3 - 15t^2 + 18t$$

(a) Velocity and Acceleration:

Velocity is the derivative of position:

$$v(t) = s'(t) = 9t^2 - 30t + 18$$

Acceleration is the derivative of velocity:

$$a(t) = v'(t) = 18t - 30$$

(b) Particle at Rest:

Set velocity to zero:

$$9t^2 - 30t + 18 = 0 \Rightarrow t = \frac{30 \pm \sqrt{(-30)^2 - 4(9)(18)}}{2(9)} = \frac{30 \pm \sqrt{900 - 648}}{18} = \frac{30 \pm \sqrt{252}}{18}$$

$$\sqrt{252} = 6\sqrt{7}, \quad t = \frac{30 \pm 6\sqrt{7}}{18} = \frac{5 \pm \sqrt{7}}{3}$$

So the particle is at rest at:

$$t = \frac{5 \pm \sqrt{7}}{3}$$

(c) Particle Moving Forward:

Forward motion means $v(t) > 0$. Use sign chart for:

$$v(t) = 9t^2 - 30t + 18 = 9(t - t_1)(t - t_2), \quad \text{where } t_1 = \frac{5 - \sqrt{7}}{3}, t_2 = \frac{5 + \sqrt{7}}{3}$$

This quadratic opens upward (coefficient of t^2 is positive), so:

$$v(t) > 0 \quad \text{on } (0, t_1) \cup (t_2, \infty)$$

So the particle moves forward on:

$$\left(0, \frac{5 - \sqrt{7}}{3}\right) \cup \left(\frac{5 + \sqrt{7}}{3}, \infty\right)$$

(d) Acceleration Zero:

$$a(t) = 18t - 30 = 0 \Rightarrow t = \frac{30}{18} = \frac{5}{3}$$

This is when the particle changes concavity or when velocity changes most rapidly.

Conclusion:

$$a(t) = 0 \text{ when } t = \frac{5}{3} \text{ seconds}$$