

Question 2

A rock is thrown vertically upward from the top of a cliff. Its height in meters after t seconds is given by:

$$h(t) = -5t^2 + 20t + 50$$

- (a) What is the velocity of the rock after 2 seconds?
- (b) When does the rock reach its maximum height?
- (c) When does the rock hit the ground?
- (d) What is the velocity of the rock at the moment it hits the ground?

Question 2 - Solution

We are given:

$$h(t) = -5t^2 + 20t + 50$$

(a) Velocity after 2 seconds:

Velocity is the derivative of height:

$$v(t) = h'(t) = -10t + 20$$

So,

$$v(2) = -10(2) + 20 = -20 + 20 = \boxed{0 \text{ m/s}}$$

(b) Maximum Height:

Maximum height occurs when velocity is zero:

$$v(t) = -10t + 20 = 0 \Rightarrow t = 2 \text{ seconds}$$

(c) When does the rock hit the ground?

The rock hits the ground when $h(t) = 0$:

$$-5t^2 + 20t + 50 = 0 \Rightarrow t^2 - 4t - 10 = 0 \Rightarrow t = \frac{4 \pm \sqrt{16 + 40}}{2} = \frac{4 \pm \sqrt{56}}{2} = \frac{4 \pm 2\sqrt{14}}{2} = 2 \pm \sqrt{14}$$

Since time must be positive, we choose the positive root:

$$t = 2 + \sqrt{14} \approx \boxed{5.74 \text{ seconds}}$$

(d) Velocity at impact:

Plug into the velocity function:

$$v(2 + \sqrt{14}) = -10(2 + \sqrt{14}) + 20 = -20 - 10\sqrt{14} + 20 = -10\sqrt{14}$$

So the velocity at impact is:

$$\boxed{-10\sqrt{14} \text{ m/s}} \quad (\text{downward})$$