

Question 1

The position of a particle moving along a straight line is given by

$$s(t) = t^3 - 6t^2 + 9t \quad \text{for } t \geq 0$$

- (a) Find the velocity and acceleration functions.
- (b) At what time(s) is the particle at rest?
- (c) Determine when the particle is speeding up and when it is slowing down.

Question 1 - Solution

We are given the position function:

$$s(t) = t^3 - 6t^2 + 9t$$

(a) Velocity and Acceleration:

Velocity is the first derivative of position:

$$v(t) = s'(t) = 3t^2 - 12t + 9$$

Acceleration is the derivative of velocity:

$$a(t) = v'(t) = 6t - 12$$

(b) Particle at Rest:

The particle is at rest when $v(t) = 0$:

$$3t^2 - 12t + 9 = 0 \Rightarrow t^2 - 4t + 3 = 0 \Rightarrow (t - 1)(t - 3) = 0 \Rightarrow t = 1, 3$$

So, the particle is at rest at $t = \boxed{1}$ and $t = \boxed{3}$.

(c) Speeding Up / Slowing Down:

Speed increases when velocity and acceleration have the same sign; slows down when signs differ.

Critical points from previous steps:

$$v(t) = 3t^2 - 12t + 9, \quad a(t) = 6t - 12$$

Velocity is 0 at $t = 1, 3$, acceleration is 0 at $t = 2$

Test signs in intervals:

$t < 1$: $v(t) > 0$, $a(t) < 0 \rightarrow$ slowing down

$1 < t < 2$: $v(t) < 0$, $a(t) < 0 \rightarrow$ speeding up

$2 < t < 3$: $v(t) < 0$, $a(t) > 0 \rightarrow$ slowing down

$t > 3$: $v(t) > 0$, $a(t) > 0 \rightarrow$ speeding up

Conclusion:

- Speeding up: $t \in (1, 2) \cup (3, \infty)$
- Slowing down: $t \in (0, 1) \cup (2, 3)$