

Question 10

Let

$$y = \frac{(x^2 + 1)^5 \cdot \sqrt[3]{x^2 - 4}}{\sqrt{2x + 1} \cdot (x - 3)^2}$$

- (a) Use logarithmic differentiation to find $\frac{dy}{dx}$.
- (b) Simplify your answer fully.

Question 10 - Solution

The real domain is $x > -\frac{1}{2}$, $x \neq 3$. Where the function is nonzero, use absolute values in logarithmic differentiation:

$$\ln |y| = 5 \ln(x^2 + 1) + \frac{1}{3} \ln |x^2 - 4| - \frac{1}{2} \ln(2x + 1) - 2 \ln |x - 3|$$

Differentiating gives

$$y' = y \left(\frac{10x}{x^2 + 1} + \frac{2x}{3(x^2 - 4)} - \frac{1}{2x + 1} - \frac{2}{x - 3} \right).$$

This formula holds for $x > -\frac{1}{2}$, $x \neq 2, 3$. At $x = 2$, $y = 0$, but there is no finite derivative because of the cube-root factor.