

Question 9

Let

$$y = \frac{\sqrt{(x^2 + 4)^3(x - 1)^2}}{(x^2 - 1)^4}$$

- (a) Use logarithmic differentiation to find $\frac{dy}{dx}$.
- (b) Simplify your final answer fully.

Question 9 - Solution

For $x \neq \pm 1$, the square root simplifies to

$$y = \frac{(x^2 + 4)^{3/2}|x - 1|}{(x^2 - 1)^4} > 0.$$

Hence

$$\ln y = \frac{3}{2} \ln(x^2 + 4) + \ln|x - 1| - 4 \ln|x^2 - 1|.$$

Differentiating and multiplying by y gives

$$y' = \frac{(x^2 + 4)^{3/2}|x - 1|}{(x^2 - 1)^4} \left(\frac{3x}{x^2 + 4} + \frac{1}{x - 1} - \frac{8x}{x^2 - 1} \right), \quad x \neq \pm 1.$$