

Question 8

Let the function be:

$$y = (x^2 + 3x + 1)^4 \cdot \sqrt{x^2 - 1} \cdot \frac{1}{(2x + 5)^3}$$

- (a) Use logarithmic differentiation to find $\frac{dy}{dx}$.
- (b) Simplify your answer as much as possible.

Question 8 - Solution

Put $A = x^2 + 3x + 1$. The real domain is $|x| \geq 1$, $x \neq -5/2$.

For $A \neq 0$ and $|x| > 1$, logarithmic differentiation gives

$$\ln |y| = 4 \ln |A| + \frac{1}{2} \ln(x^2 - 1) - 3 \ln |2x + 5|,$$

$$\frac{y'}{y} = \frac{4(2x + 3)}{A} + \frac{x}{x^2 - 1} - \frac{6}{2x + 5}.$$

Cancel before evaluating at zeros of A :

$$y' = \frac{A^3 [4(2x + 3)(x^2 - 1)(2x + 5) + Ax(2x + 5) - 6A(x^2 - 1)]}{\sqrt{x^2 - 1}(2x + 5)^4}.$$

This formula holds for $|x| > 1$, $x \neq -5/2$, including $x = (-3 - \sqrt{5})/2$, where $y' = 0$. The other root of A is outside the domain.

There is no finite derivative at $x = \pm 1$.