

Question 7

Let the function be:

$$y = \left(\frac{(2x^2 + 1)^5}{\sqrt{5x - 3} \cdot (x^2 + 4)^3} \right)^2$$

- (a) Use logarithmic differentiation to find $\frac{dy}{dx}$.
- (b) Simplify the result as much as possible.

Question 7 - Solution

We are given:

$$y = \left(\frac{(2x^2 + 1)^5}{\sqrt{5x - 3} \cdot (x^2 + 4)^3} \right)^2$$

Take the natural logarithm of both sides:

$$\ln y = \ln \left[\left(\frac{(2x^2 + 1)^5}{(5x - 3)^{1/2} \cdot (x^2 + 4)^3} \right)^2 \right]$$

Bring the exponent outside:

$$\ln y = 2 \cdot \ln \left(\frac{(2x^2 + 1)^5}{(5x - 3)^{1/2} \cdot (x^2 + 4)^3} \right)$$

Use log rules:

$$\ln y = 2 \left[5 \ln(2x^2 + 1) - \frac{1}{2} \ln(5x - 3) - 3 \ln(x^2 + 4) \right]$$

Differentiate both sides:

$$\frac{1}{y} \cdot \frac{dy}{dx} = 2 \left[\frac{5 \cdot 4x}{2x^2 + 1} - \frac{1}{2} \cdot \frac{5}{5x - 3} - \frac{3 \cdot 2x}{x^2 + 4} \right]$$

Simplify:

$$\frac{dy}{dx} = y \cdot \left(\frac{40x}{2x^2 + 1} - \frac{5}{5x - 3} - \frac{12x}{x^2 + 4} \right)$$

Substitute y back:

$$\frac{dy}{dx} = \left(\frac{(2x^2 + 1)^5}{\sqrt{5x - 3} \cdot (x^2 + 4)^3} \right)^2 \cdot \left(\frac{40x}{2x^2 + 1} - \frac{5}{5x - 3} - \frac{12x}{x^2 + 4} \right)$$

Final Answer:

$$\boxed{\frac{dy}{dx} = \left(\frac{(2x^2 + 1)^5}{\sqrt{5x - 3} \cdot (x^2 + 4)^3} \right)^2 \cdot \left(\frac{40x}{2x^2 + 1} - \frac{5}{5x - 3} - \frac{12x}{x^2 + 4} \right)}$$