

Question 6

Let the function be:

$$f(x) = \frac{(x^2 + 3x + 5)^4 \cdot \sqrt{x^2 - 1}}{(x + 2)^5 \cdot (3x - 1)^{1/3}}$$

- (a) Use logarithmic differentiation to find $f'(x)$.
- (b) Simplify the result as much as possible.

Question 6 - Solution

The real domain is $|x| \geq 1$, $x \neq -2$. Where the function is nonzero, use absolute values in logarithmic differentiation:

$$\ln |f(x)| = 4 \ln(x^2 + 3x + 5) + \frac{1}{2} \ln(x^2 - 1) - 5 \ln |x + 2| - \frac{1}{3} \ln |3x - 1|$$

Differentiating gives

$$f'(x) = f(x) \left(\frac{4(2x+3)}{x^2+3x+5} + \frac{x}{x^2-1} - \frac{5}{x+2} - \frac{1}{3x-1} \right).$$

This formula holds for $|x| > 1$, $x \neq -2$. At $x = \pm 1$ there is no finite derivative. The cube root is the real cube root.