

**Question 5**

Let the function be:

$$f(x) = \frac{(x^2 + 1)^5 \cdot \sqrt[3]{\ln(x)}}{e^{x^3} \cdot (x - 4)^2}$$

- (a) Use logarithmic differentiation to find  $f'(x)$ .
- (b) Simplify your answer as much as possible.

**Question 5 - Solution**

The real domain is  $x > 0$ ,  $x \neq 4$ . Where the function is nonzero, use absolute values in logarithmic differentiation:

$$\ln |f(x)| = 5 \ln(x^2 + 1) + \frac{1}{3} \ln |\ln x| - x^3 - 2 \ln |x - 4|$$

Differentiating gives

$$f'(x) = f(x) \left( \frac{10x}{x^2 + 1} + \frac{1}{3x \ln x} - 3x^2 - \frac{2}{x - 4} \right).$$

This formula holds for  $x > 0$ ,  $x \neq 1, 4$ . At  $x = 1$ ,  $f(1) = 0$ , but the cube-root factor gives no finite derivative.