

Question 4

For $x > 1$ (so the base is positive), let the function be:

$$f(x) = \left(\frac{x^2 \cdot \sqrt{x+1}}{(x^2+4)^3 \cdot \ln x} \right)^{\sin x}$$

- (a) Use logarithmic differentiation to find $f'(x)$.
- (b) Simplify your answer as much as possible.

Question 4 - Solution

For $x > 1$, define the positive base

$$B(x) = \frac{x^2\sqrt{x+1}}{(x^2+4)^3 \ln x}, \quad f(x) = B(x)^{\sin x}.$$

Set

$$A(x) = \ln B(x) = 2 \ln x + \frac{1}{2} \ln(x+1) - 3 \ln(x^2+4) - \ln(\ln x).$$

Then $\ln f(x) = \sin x A(x)$ and

$$A'(x) = \frac{2}{x} + \frac{1}{2(x+1)} - \frac{6x}{x^2+4} - \frac{1}{x \ln x}.$$

Logarithmic differentiation yields

$$\boxed{f'(x) = B(x)^{\sin x} (\cos x A(x) + \sin x A'(x)), \quad x > 1.}$$

Here B, A, A' are the explicit expressions above.