

Question 3

Let the function be:

$$f(x) = \frac{(3x+1)^4 \cdot \ln(x)}{(x^2-9)^{3/2}}$$

- (a) Use logarithmic differentiation to find $f'(x)$.
- (b) Clearly indicate how you apply the logarithmic properties before differentiating.

Question 3 - Solution

We are given:

$$f(x) = \frac{(3x+1)^4 \cdot \ln(x)}{(x^2-9)^{3/2}}$$

Step 1: Take the natural log of both sides:

$$\ln f(x) = \ln \left(\frac{(3x+1)^4 \cdot \ln(x)}{(x^2-9)^{3/2}} \right)$$

Apply logarithmic identities:

$$\ln f(x) = 4 \ln(3x+1) + \ln(\ln x) - \frac{3}{2} \ln(x^2-9)$$

Step 2: Differentiate both sides implicitly:

$$\frac{f'(x)}{f(x)} = \frac{4 \cdot 3}{3x+1} + \frac{1}{\ln x} \cdot \frac{1}{x} - \frac{3}{2} \cdot \frac{2x}{x^2-9}$$

Simplify:

$$\frac{f'(x)}{f(x)} = \frac{12}{3x+1} + \frac{1}{x \ln x} - \frac{3x}{x^2-9}$$

Step 3: Multiply both sides by $f(x)$ to isolate $f'(x)$:

$$f'(x) = f(x) \left(\frac{12}{3x+1} + \frac{1}{x \ln x} - \frac{3x}{x^2-9} \right)$$

Substitute back the original $f(x)$:

$$f'(x) = \frac{(3x+1)^4 \cdot \ln(x)}{(x^2-9)^{3/2}} \left(\frac{12}{3x+1} + \frac{1}{x \ln x} - \frac{3x}{x^2-9} \right)$$

$$\boxed{f'(x) = \frac{(3x+1)^4 \cdot \ln(x)}{(x^2-9)^{3/2}} \left(\frac{12}{3x+1} + \frac{1}{x \ln x} - \frac{3x}{x^2-9} \right)}$$