

Question 2

Let the function be:

$$f(x) = \frac{(x^2 + 1)^5 \cdot \sqrt{x + 3}}{(x - 1)^4}$$

- (a) Use logarithmic differentiation to find $f'(x)$.
- (b) Simplify your final expression as much as possible.

Question 2 - Solution

The real domain is $x \geq -3$, $x \neq 1$. Where the function is nonzero, use absolute values in logarithmic differentiation:

$$\ln |f(x)| = 5 \ln(x^2 + 1) + \frac{1}{2} \ln(x + 3) - 4 \ln |x - 1|$$

Differentiating gives

$$f'(x) = f(x) \left(\frac{10x}{x^2 + 1} + \frac{1}{2(x + 3)} - \frac{4}{x - 1} \right).$$

This formula holds for $x > -3$, $x \neq 1$. At $x = -3$ there is no finite derivative because of the square-root factor.