

Question 1

Use logarithmic differentiation to find the derivative of:

$$f(x) = (x^2 + 1)^x$$

Simplify your answer as much as possible.

Question 1 - Solution

We are given:

$$f(x) = (x^2 + 1)^x$$

Take the natural log of both sides:

$$\ln f(x) = \ln((x^2 + 1)^x) = x \ln(x^2 + 1)$$

Differentiate both sides implicitly:

$$\frac{1}{f(x)} \cdot f'(x) = \ln(x^2 + 1) + x \cdot \frac{1}{x^2 + 1} \cdot 2x = \ln(x^2 + 1) + \frac{2x^2}{x^2 + 1}$$

Multiply both sides by $f(x) = (x^2 + 1)^x$:

$$f'(x) = (x^2 + 1)^x \left[\ln(x^2 + 1) + \frac{2x^2}{x^2 + 1} \right]$$

Final Answer:

$$f'(x) = \boxed{(x^2 + 1)^x \left[\ln(x^2 + 1) + \frac{2x^2}{x^2 + 1} \right]}$$