

**Question 9**

Let  $f(x) = \cos(2x)$ .

- (a) Find the first four derivatives of  $f(x)$ .
- (b) Find a general formula for the  $n$ -th derivative  $f^{(n)}(x)$ .
- (c) Evaluate  $f^{(15)}(x)$ .

**Question 9 - Solution**

Successive differentiation yields

$$f' = -2 \sin 2x, \quad f'' = -4 \cos 2x, \quad f^{(3)} = 8 \sin 2x, \quad f^{(4)} = 16 \cos 2x.$$

Each derivative multiplies by 2 and advances the phase by  $\pi/2$ ; induction therefore gives

$$\boxed{f^{(n)}(x) = 2^n \cos(2x + n\pi/2)}.$$

Since  $15\pi/2$  is congruent to  $3\pi/2$  modulo  $2\pi$ ,

$$\boxed{f^{(15)}(x) = 32768 \cos(2x + 3\pi/2) = 32768 \sin 2x}.$$