

Question 8

Let $f(x) = \frac{1}{x}$.

- (a) Find the first four derivatives of $f(x)$.
- (b) Based on the pattern, write a general formula for the n -th derivative $f^{(n)}(x)$.
- (c) Evaluate $f^{(6)}(x)$.

Question 8 - Solution

We are given:

$$f(x) = \frac{1}{x} = x^{-1}$$

(a) Compute derivatives:

$$f'(x) = \frac{d}{dx}(x^{-1}) = -x^{-2}$$

$$f''(x) = \frac{d}{dx}(-x^{-2}) = 2x^{-3}$$

$$f^{(3)}(x) = \frac{d}{dx}(2x^{-3}) = -6x^{-4}$$

$$f^{(4)}(x) = \frac{d}{dx}(-6x^{-4}) = 24x^{-5}$$

(b) General Pattern:

Observe the signs and coefficients:

$$f^{(n)}(x) = (-1)^n \cdot n! \cdot x^{-(n+1)}$$

Answer: $f^{(n)}(x) = (-1)^n \cdot n! \cdot x^{-(n+1)}$

(c) Evaluate $f^{(6)}(x)$:

Use the formula:

$$f^{(6)}(x) = (-1)^6 \cdot 6! \cdot x^{-7} = 720x^{-7}$$

Answer: $f^{(6)}(x) = \frac{720}{x^7}$