

Question 1

Let:

$$f(x) = x^4 - 4x^3 + 6x^2$$

- (a) Find the first and second derivatives of $f(x)$.
- (b) Determine the intervals on which $f(x)$ is concave up and concave down.
- (c) Find all points of inflection.

Question 1 - Solution

We are given:

$$f(x) = x^4 - 4x^3 + 6x^2$$

(a) First derivative:

$$f'(x) = 4x^3 - 12x^2 + 12x$$

Second derivative:

$$f''(x) = 12x^2 - 24x + 12$$

Factor:

$$f''(x) = 12(x^2 - 2x + 1) = 12(x - 1)^2$$

Answer:

$$f'(x) = 4x^3 - 12x^2 + 12x, \quad f''(x) = 12(x - 1)^2$$

(b) Analyze sign of $f''(x)$:

Since $f''(x) = 12(x - 1)^2 \geq 0$ for all x , the second derivative is never negative.

$f''(x) > 0$ for $x \neq 1 \rightarrow$ concave up

$f''(1) = 0 \rightarrow$ possible inflection point

But since the sign does not change around $x = 1$, it is not an inflection point.

Answer: Concave up: $(-\infty, \infty)$ Concave down: None

(c) No change in concavity at $x = 1$, so:

Answer: No points of inflection