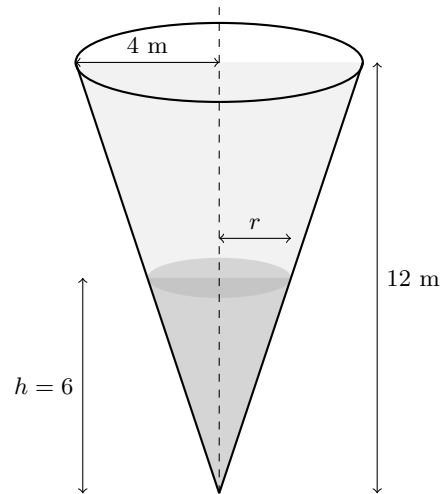


Question 10

Water is poured into an inverted conical tank at a rate of $3 \text{ m}^3/\text{min}$. The tank is 12 meters deep and has a top radius of 4 meters.

How fast is the water level rising when the water is 6 meters deep?



Question 10 - Solution

Let:

V : volume of water

r : radius of water surface

h : depth of water

Given:

$$\frac{dV}{dt} = 3 \text{ m}^3/\text{min}$$

Tank dimensions: height = 12 m, top radius = 4 m

$$\text{Volume of cone: } V = \frac{1}{3}\pi r^2 h$$

Use similar triangles:

$$\frac{r}{h} = \frac{4}{12} = \frac{1}{3} \Rightarrow r = \frac{h}{3}$$

Substitute into volume formula:

$$V = \frac{1}{3}\pi \left(\frac{h}{3}\right)^2 h = \frac{\pi}{27}h^3$$

Differentiate:

$$\frac{dV}{dt} = \frac{\pi}{9}h^2 \cdot \frac{dh}{dt}$$

Substitute known values:

$$3 = \frac{\pi}{9}(6)^2 \cdot \frac{dh}{dt} = \frac{\pi}{9} \cdot 36 \cdot \frac{dh}{dt} \Rightarrow 3 = 4\pi \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{3}{4\pi}$$

Final Answer:

$$\boxed{\frac{dh}{dt} = \frac{3}{4\pi} \text{ m/min} \approx 0.239 \text{ m/min}}$$