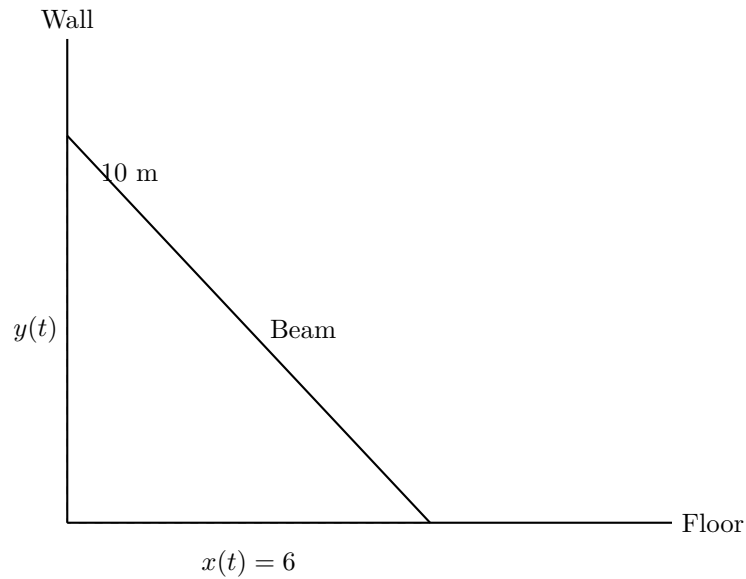


Question 9

A 10-meter-long beam is leaning against a vertical wall and a horizontal floor, forming a right triangle. The bottom of the beam is sliding away from the wall along the floor at a rate of 1.5 m/s.

How fast is the top of the beam sliding down the wall when the bottom is 6 meters away from the wall?



Question 9 - Solution

Let:

$x(t)$: distance from the base of the wall to the bottom of the beam

$y(t)$: height of the beam on the wall

The beam is 10 m long $\Rightarrow x^2 + y^2 = 100$

Differentiate both sides with respect to time:

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \Rightarrow x \frac{dx}{dt} + y \frac{dy}{dt} = 0$$

Given: $x = 6$, $\frac{dx}{dt} = 1.5$

Find y from Pythagoras:

$$y = \sqrt{100 - 6^2} = \sqrt{64} = 8$$

Now solve for $\frac{dy}{dt}$:

$$6(1.5) + 8 \frac{dy}{dt} = 0 \Rightarrow 9 + 8 \frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{9}{8}$$

Conclusion: The top of the beam is sliding down at a rate of:

$\frac{dy}{dt} = -\frac{9}{8} = -1.125 \text{ m/s}$
