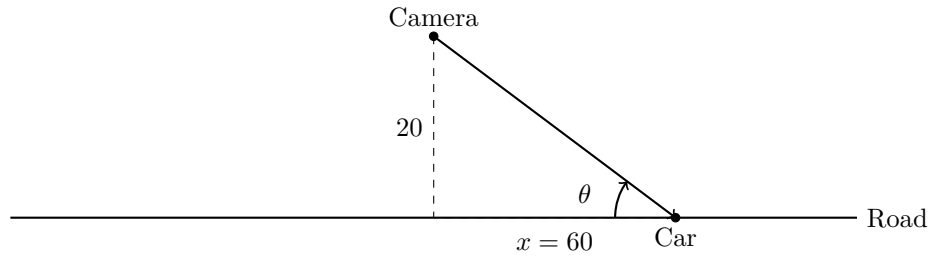


Question 8

A camera is placed 20 meters from a straight road. A car is driving along the road away from the point closest to the camera at a speed of 30 m/s.

How fast is the angle between the camera's line of sight and the road changing when the car is 60 meters from the point on the road closest to the camera?



Question 8 - Solution

Let:

$x(t)$: horizontal distance from car to closest point on road to camera , $\theta(t)$: angle between the camera's line of sight and the road , The camera is 20 m from the road (fixed)

We have:

$$\tan(\theta) = \frac{20}{x}$$

Differentiate both sides with respect to time:

$$\sec^2(\theta) \cdot \frac{d\theta}{dt} = \frac{-20}{x^2} \cdot \frac{dx}{dt}$$

At the instant:

$$x = 60, \frac{dx}{dt} = 30 \text{ m/s (moving away)}, \tan(\theta) = \frac{20}{60} = \frac{1}{3} \Rightarrow \theta \approx 18.43^\circ$$

Use identity:

$$\sec^2(\theta) = 1 + \tan^2(\theta) = 1 + \left(\frac{1}{3}\right)^2 = \frac{10}{9}$$

Now plug in:

$$\frac{10}{9} \cdot \frac{d\theta}{dt} = \frac{-20}{60^2} \cdot 30 = \frac{-20 \cdot 30}{3600} = \frac{-600}{3600} = -\frac{1}{6}$$

So:

$$\frac{d\theta}{dt} = \boxed{-\frac{9}{60} = -0.15 \text{ radians per second}}$$

Conclusion: The angle between the camera's line of sight and the road is decreasing at a rate of:

$$\boxed{0.15 \text{ radians/second}}$$