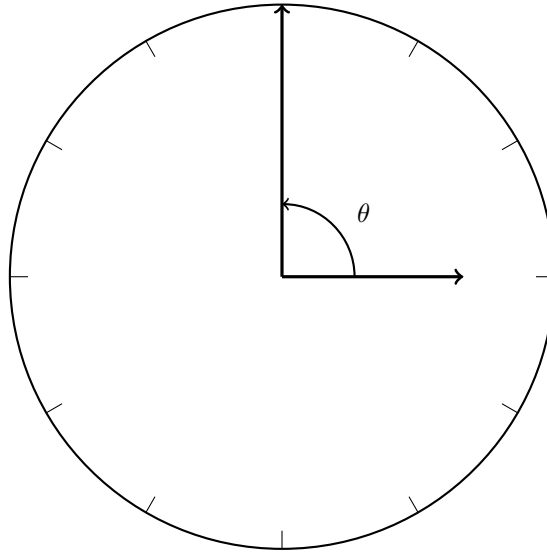


Question 7

The minute hand of a clock completes one full revolution every 60 minutes, and the hour hand completes one full revolution every 12 hours.

Question: At exactly 3:00, how fast is the *smaller angle* between the hour and minute hands changing?



Solution — Method 1 (Formal Related Rates)

Let $\theta(t)$ be the signed angle (in degrees) from the hour hand to the minute hand.

$$\omega_m = 360/60 = 6^\circ/\text{min}, \quad \omega_h = 360/(12 * 60) = 0.5^\circ/\text{min}$$

At 3:00, the hour hand is at 90° and the minute hand is at 0° .

$$\theta(t) = (6t) - (90 + 0.5t)$$

Differentiate:

$$\frac{d\theta}{dt} = 6 - 0.5 = 5.5$$

Since the smaller angle is initially decreasing,

$$\boxed{\frac{d\theta}{dt} = -5.5^\circ/\text{min}}$$

Question 7 - Solution)

The angle between the hands changes at the difference of their angular speeds.

$$\text{Minute hand speed} = 6^\circ/\text{min}$$

$$\text{Hour hand speed} = 0.5^\circ/\text{min}$$

So the angle closes at:

$$6 - 0.5 = 5.5^\circ/\text{min}$$

Since the hands are moving toward each other at 3:00, the angle is decreasing.

$$\boxed{5.5^\circ/\text{min (decreasing)}}$$

Final Answer

$$\boxed{\frac{d\theta}{dt} = -5.5^\circ/\text{min}}$$

The negative sign indicates the angle between the hands is decreasing.