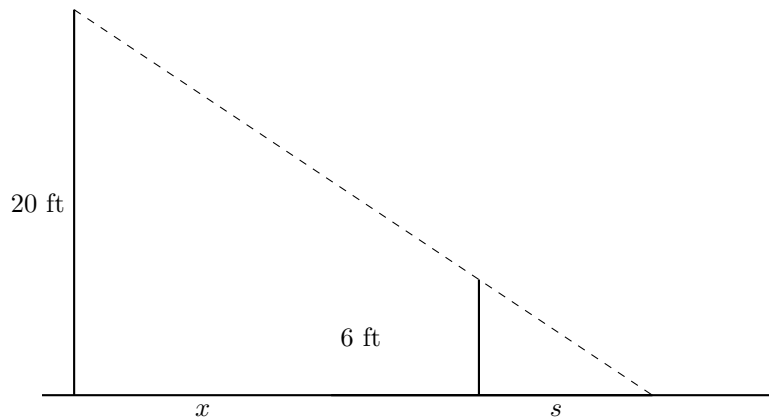


Question 1

A streetlight is mounted at the top of a 20-foot pole. A person 6 feet tall walks directly away from the pole at a constant rate of 4 ft/sec.

- (a) How fast is the tip of the person's shadow moving along the ground when the person is 10 feet from the pole?
- (b) How fast is the length of the person's shadow changing at that instant?
- (c) Explain why the tip of the shadow moves faster than the person.



Question 1 - Solution

Let:

$x(t)$ = distance from the pole to the person

$s(t)$ = length of the shadow

The distance from the pole to the tip of the shadow is:

$$x + s$$

Step 1: Set up the relationship

Using similar triangles:

$$\frac{20}{x + s} = \frac{6}{s}$$

Cross-multiply:

$$20s = 6(x + s)$$

Simplify:

$$20s = 6x + 6s \Rightarrow 14s = 6x \Rightarrow s = \frac{3}{7}x$$

Step 2: Differentiate with respect to time

Differentiate both sides:

$$\frac{ds}{dt} = \frac{3}{7} \frac{dx}{dt}$$

Given:

$$\frac{dx}{dt} = 4$$

So:

$$\frac{ds}{dt} = \frac{12}{7}$$

(a) Speed of the tip of the shadow

$$\frac{d}{dt}(x + s) = \frac{dx}{dt} + \frac{ds}{dt} = 4 + \frac{12}{7} = \frac{40}{7}$$

Answer (a):

$$\boxed{\frac{40}{7} \text{ ft/sec}}$$

(b) Rate of change of the shadow length

$$\boxed{\frac{ds}{dt} = \frac{12}{7} \text{ ft/sec}}$$

(c) Explanation

The shadow tip moves faster than the person because its motion includes both the person's movement and the changing length of the shadow.

Conclusion: The shadow tip moves faster than the person.