

Question 10

Consider the real relation

$$x^2 + y^2 = \sin(xy).$$

- (a) Derive the formal relation obtained by implicit differentiation. State when division would be valid.
- (b) Determine the real solution set and decide whether a tangent slope exists at $(0, 0)$.

Question 10 - Solution

Formal differentiation. If a differentiable branch existed, it would obey

$$2x + 2yy' = \cos(xy)(y + xy'),$$

so $(2y - x \cos(xy))y' = y \cos(xy) - 2x$. Division would require a nonzero coefficient.

Analyze the real point set first. For every real x, y ,

$$\sin(xy) \leq |xy| \leq \frac{x^2 + y^2}{2}.$$

Thus the given equation implies

$$0 \leq x^2 + y^2 \leq \frac{x^2 + y^2}{2},$$

which forces $x = y = 0$. The real solution set is the singleton $\boxed{\{(0, 0)\}}$.

There is no curve branch through this isolated point and hence no tangent slope dy/dx there. The formal expression $0/0$ alone would not establish this conclusion.