

Question 9

Consider the equation:

$$x \sin(y) + y \cos(x) = 1$$

- (a) Use implicit differentiation to find $\frac{dy}{dx}$.
- (b) Find the slope of the tangent line to the curve at the point $(0, 1)$.

Question 9 - Solution

We are given:

$$x \sin(y) + y \cos(x) = 1$$

Differentiate both sides implicitly with respect to x :

$$\frac{d}{dx}[x \sin(y)] + \frac{d}{dx}[y \cos(x)] = 0$$

Using the product rule:

$$\sin(y) + x \cos(y) \frac{dy}{dx} + \cos(x) \frac{dy}{dx} - y \sin(x) = 0$$

Move the non- $\frac{dy}{dx}$ terms to the other side:

$$x \cos(y) \frac{dy}{dx} + \cos(x) \frac{dy}{dx} = y \sin(x) - \sin(y)$$

Factor out $\frac{dy}{dx}$:

$$(x \cos(y) + \cos(x)) \frac{dy}{dx} = y \sin(x) - \sin(y)$$

Solve for $\frac{dy}{dx}$:

$$\boxed{\frac{dy}{dx} = \frac{y \sin(x) - \sin(y)}{x \cos(y) + \cos(x)}}$$

At $(0, 1)$:

$$\frac{dy}{dx} = \frac{(1) \sin(0) - \sin(1)}{(0) \cos(1) + \cos(0)}$$

$$\frac{dy}{dx} = \frac{0 - \sin(1)}{0 + 1} = \boxed{-\sin(1)}$$

Therefore, the slope of the tangent line at $(0, 1)$ is:

$$\boxed{-\sin(1)}$$