

Question 7

Given the curve defined implicitly by the equation:

$$\sin(xy) + x^2 = y$$

- (a) Use implicit differentiation to find $\frac{dy}{dx}$.
- (b) Find the value of $\frac{dy}{dx}$ at the point $(0, 0)$.

Question 7 - Solution

We are given:

$$\sin(xy) + x^2 = y$$

(a) Differentiate both sides implicitly with respect to x :

Use the chain rule on $\sin(xy)$, noting that xy is a product of functions:

$$\frac{d}{dx}[\sin(xy)] + \frac{d}{dx}[x^2] = \frac{d}{dx}[y]$$

Differentiate:

$$\cos(xy) \cdot \left(y + x \cdot \frac{dy}{dx} \right) + 2x = \frac{dy}{dx}$$

Now solve for $\frac{dy}{dx}$. Expand the left-hand side:

$$\cos(xy)y + \cos(xy)x \cdot \frac{dy}{dx} + 2x = \frac{dy}{dx}$$

Bring all $\frac{dy}{dx}$ terms to one side:

$$\cos(xy)x \cdot \frac{dy}{dx} - \frac{dy}{dx} = -\cos(xy)y - 2x$$

Factor $\frac{dy}{dx}$:

$$(\cos(xy)x - 1) \cdot \frac{dy}{dx} = -\cos(xy)y - 2x$$

Solve:

$$\boxed{\frac{dy}{dx} = \frac{-\cos(xy)y - 2x}{\cos(xy)x - 1}}$$

(b) Evaluate at the point $(0, 0)$:

Substitute $x = 0$, $y = 0$ into the derivative:

$$\cos(0) = 1$$

$$\frac{dy}{dx} = \frac{-1(0) - 2(0)}{1(0) - 1} = \frac{0}{-1} = 0$$

$$\boxed{\left. \frac{dy}{dx} \right|_{(0,0)} = 0}$$