

Question 5

Consider the equation:

$$x^2y + e^{xy} = y^3 + 4x$$

- (a) Use implicit differentiation to find $\frac{dy}{dx}$.
- (b) Find the slope of the tangent line to the curve at the point $(0, 1)$.

Question 5 - Solution

We are given:

$$x^2y + e^{xy} = y^3 + 4x$$

Differentiate both sides implicitly with respect to x :

$$\frac{d}{dx}[x^2y] + \frac{d}{dx}[e^{xy}] = \frac{d}{dx}[y^3] + \frac{d}{dx}[4x]$$

Using the product rule and chain rule:

$$x^2 \frac{dy}{dx} + 2xy + e^{xy} \left(x \frac{dy}{dx} + y \right) = 3y^2 \frac{dy}{dx} + 4$$

Expand:

$$x^2 \frac{dy}{dx} + 2xy + xe^{xy} \frac{dy}{dx} + ye^{xy} = 3y^2 \frac{dy}{dx} + 4$$

Move the $\frac{dy}{dx}$ terms to one side:

$$x^2 \frac{dy}{dx} + xe^{xy} \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 4 - 2xy - ye^{xy}$$

Factor:

$$(x^2 + xe^{xy} - 3y^2) \frac{dy}{dx} = 4 - 2xy - ye^{xy}$$

Therefore,

$$\boxed{\frac{dy}{dx} = \frac{4 - 2xy - ye^{xy}}{x^2 + xe^{xy} - 3y^2}}$$

At $(0, 1)$, we have $xy = 0$, so $e^{xy} = e^0 = 1$. Thus,

$$\frac{dy}{dx} = \frac{4 - 2(0)(1) - (1)(1)}{0^2 + 0(1) - 3(1)^2} = \frac{3}{-3} = \boxed{-1}$$

Therefore, the slope of the tangent line at $(0, 1)$ is:

$$\boxed{-1}$$