

Question 4

Consider the equation:

$$\sin(xy) = x + y$$

- (a) Use implicit differentiation to find $\frac{dy}{dx}$.
- (b) Find the slope of the tangent line to the curve at the point $(0, 0)$.

Question 4 - Solution**(b) Tangent slope**

At $(0, 0)$, the original equation reads $\sin 0 = 0 + 0$, so the point belongs to the curve.

(a) Implicit derivative

Implicit differentiation gives

$$\cos(xy)(y + xy') = 1 + y'.$$

Collecting derivative terms,

$$(x \cos(xy) - 1)y' = 1 - y \cos(xy).$$

Thus, wherever the denominator is nonzero,

$$y' = \frac{1 - y \cos(xy)}{x \cos(xy) - 1}.$$

At the specified point the denominator is -1 , and the slope is $\boxed{-1}$.