

Question 3

Consider the equation:

$$x^2 \sin(y) + y^2 \cos(x) = 1$$

- (a) Use implicit differentiation to find $\frac{dy}{dx}$.
- (b) Find the slope of the tangent line to the curve at the point $(0, 1)$.

Question 3 - Solution

We are given:

$$x^2 \sin(y) + y^2 \cos(x) = 1$$

Differentiate both sides implicitly with respect to x :

$$\frac{d}{dx} [x^2 \sin(y)] + \frac{d}{dx} [y^2 \cos(x)] = 0$$

Using the product rule and chain rule:

$$2x \sin(y) + x^2 \cos(y) \frac{dy}{dx} + 2y \frac{dy}{dx} \cos(x) - y^2 \sin(x) = 0$$

Move the non- $\frac{dy}{dx}$ terms to the other side:

$$x^2 \cos(y) \frac{dy}{dx} + 2y \cos(x) \frac{dy}{dx} = y^2 \sin(x) - 2x \sin(y)$$

Factor out $\frac{dy}{dx}$:

$$(x^2 \cos(y) + 2y \cos(x)) \frac{dy}{dx} = y^2 \sin(x) - 2x \sin(y)$$

Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{y^2 \sin(x) - 2x \sin(y)}{x^2 \cos(y) + 2y \cos(x)}$$

At $(0, 1)$:

$$\frac{dy}{dx} = \frac{(1)^2 \sin(0) - 2(0) \sin(1)}{(0)^2 \cos(1) + 2(1) \cos(0)} = \frac{0}{2} = \boxed{0}$$

Therefore, the slope of the tangent line at $(0, 1)$ is:

$$\boxed{0}$$