

Question 2

Let the equation of a curve be defined implicitly by:

$$x^3 + y^3 = 6xy$$

- (a) Use implicit differentiation to find $\frac{dy}{dx}$.
- (b) Find the slope of the tangent line at the point $(3, 3)$.

Question 2 - Solution

We are given:

$$x^3 + y^3 = 6xy$$

(a) Differentiate both sides with respect to x :

Left side:

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) = 3x^2 + 3y^2 \frac{dy}{dx}$$

Right side (Product Rule):

$$\frac{d}{dx}(6xy) = 6x \frac{dy}{dx} + 6y$$

Now equate both sides:

$$3x^2 + 3y^2 \frac{dy}{dx} = 6x \frac{dy}{dx} + 6y$$

Bring like terms together:

$$3y^2 \frac{dy}{dx} - 6x \frac{dy}{dx} = 6y - 3x^2$$

Factor out $\frac{dy}{dx}$:

$$(3y^2 - 6x) \frac{dy}{dx} = 6y - 3x^2$$

Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$

Simplify:

$$\boxed{\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}}$$

(b) Evaluate the derivative at $(3, 3)$:

Substitute $x = 3$, $y = 3$:

$$\frac{dy}{dx} = \frac{2(3) - 3^2}{3^2 - 2(3)} = \frac{6 - 9}{9 - 6} = \frac{-3}{3} = \boxed{-1}$$