

Question 1

Given the equation:

$$x^2 + xy + y^2 = 7$$

- (a) Use implicit differentiation to find $\frac{dy}{dx}$.
- (b) Find the slope of the tangent line at the point $(1, 2)$.

Question 1 - Solution

We are given:

$$x^2 + xy + y^2 = 7$$

Differentiate both sides with respect to x , using the product rule on xy :

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(7)$$

$$2x + \left(x \frac{dy}{dx} + y\right) + 2y \frac{dy}{dx} = 0$$

Group $\frac{dy}{dx}$ terms:

$$x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2x - y \Rightarrow (x + 2y) \frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} = \boxed{\frac{-2x - y}{x + 2y}}$$

(b) Evaluate at the point $(1, 2)$:

$$\frac{dy}{dx} = \frac{-2(1) - 2}{1 + 2(2)} = \frac{-4}{5}$$

Answer: Slope of the tangent line at $(1, 2)$ is $\boxed{-\frac{4}{5}}$