

Question 10

Let

$$f(x) = \sqrt{x+3}$$

Use the definition of the derivative to find $f'(x)$. Do not apply any derivative shortcuts — use first principles only.

Question 10 - Solution

We are given:

$$f(x) = \sqrt{x+3}$$

Apply the definition of the derivative:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h}$$

To evaluate this limit, multiply numerator and denominator by the conjugate of the numerator:

$$= \lim_{h \rightarrow 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h} \cdot \frac{\sqrt{x+h+3} + \sqrt{x+3}}{\sqrt{x+h+3} + \sqrt{x+3}}$$

Use the identity $(a-b)(a+b) = a^2 - b^2$:

$$= \lim_{h \rightarrow 0} \frac{(x+h+3) - (x+3)}{h(\sqrt{x+h+3} + \sqrt{x+3})} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h+3} + \sqrt{x+3})}$$

Cancel h in numerator and denominator:

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h+3} + \sqrt{x+3}}$$

Now take the limit as $h \rightarrow 0$:

$$f'(x) = \frac{1}{2\sqrt{x+3}}$$

Final Answer:

$$f'(x) = \boxed{\frac{1}{2\sqrt{x+3}}}$$