

Question 8

Let

$$f(x) = \sqrt{x}$$

Use the definition of the derivative to compute $f'(x)$. Do not use derivative shortcuts. Show all algebraic simplification and steps clearly.

Question 8 - Solution

We are given:

$$f(x) = \sqrt{x}$$

Use the definition of the derivative:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

Multiply numerator and denominator by the conjugate:

$$= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \lim_{h \rightarrow 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})}$$

Simplify numerator:

$$= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

Now take the limit as $h \rightarrow 0$:

$$f'(x) = \frac{1}{2\sqrt{x}}$$

Final Answer:

$$f'(x) = \boxed{\frac{1}{2\sqrt{x}}}$$