

Question 7

Let

$$f(x) = \frac{1}{x}$$

Use the definition of the derivative to compute $f'(x)$. Show all steps clearly without using derivative rules.

Question 7 - Solution

We are given:

$$f(x) = \frac{1}{x}$$

Use the definition:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

Combine the terms in the numerator:

$$= \lim_{h \rightarrow 0} \frac{\frac{x - (x+h)}{x(x+h)}}{h} = \lim_{h \rightarrow 0} \frac{\frac{-h}{x(x+h)}}{h}$$

Simplify:

$$= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)}$$

Now take the limit as $h \rightarrow 0$:

$$f'(x) = \frac{-1}{x^2}$$

Final Answer:

$$f'(x) = \boxed{-\frac{1}{x^2}}$$