

Question 6

Let

$$f(x) = \sqrt{x+1}$$

Use the definition of the derivative to find $f'(x)$. Show all steps clearly without using derivative shortcuts.

Question 6 - Solution

We are given:

$$f(x) = \sqrt{x+1}$$

Use the definition of the derivative:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$$

To simplify the numerator, multiply by the conjugate:

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}$$

This becomes:

$$= \lim_{h \rightarrow 0} \frac{(x+h+1) - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})}$$

Cancel the h 's:

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}}$$

Now take the limit as $h \rightarrow 0$:

$$f'(x) = \frac{1}{2\sqrt{x+1}}$$

Final Answer:

$$f'(x) = \boxed{\frac{1}{2\sqrt{x+1}}}$$