

Question 4

Let

$$f(x) = x^2$$

Use the definition of the derivative to compute $f'(3)$. Do not use differentiation rules.

Question 4 - Solution

We are given:

$$f(x) = x^2$$

Using the definition of the derivative at $x = 3$:

$$f'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$$

Compute each term:

$$f(3+h) = (3+h)^2 = 9 + 6h + h^2$$

$$f(3) = 3^2 = 9$$

Substitute into the limit:

$$f'(3) = \lim_{h \rightarrow 0} \frac{(9 + 6h + h^2) - 9}{h} = \lim_{h \rightarrow 0} \frac{6h + h^2}{h}$$

Factor h from the numerator:

$$= \lim_{h \rightarrow 0} \frac{h(6 + h)}{h}$$

Cancel h :

$$= \lim_{h \rightarrow 0} (6 + h)$$

Now take the limit:

$$f'(3) = \boxed{6}$$