

Question 3

Let

$$f(x) = \frac{1}{x+3}$$

Use the definition of the derivative to compute $f'(1)$. Show all steps clearly without using differentiation shortcuts.

Question 3 - Solution

We are given:

$$f(x) = \frac{1}{x+3}$$

Using the definition of the derivative:

$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$$

Compute:

$$f(1+h) = \frac{1}{1+h+3} = \frac{1}{h+4}, \quad f(1) = \frac{1}{4}$$

So,

$$f'(1) = \lim_{h \rightarrow 0} \frac{\frac{1}{h+4} - \frac{1}{4}}{h}$$

Combine the terms in the numerator:

$$= \lim_{h \rightarrow 0} \frac{\frac{4-(h+4)}{4(h+4)}}{h} = \lim_{h \rightarrow 0} \frac{\frac{-h}{4(h+4)}}{h}$$

Simplify:

$$= \lim_{h \rightarrow 0} \frac{-1}{4(h+4)}$$

Now take the limit:

$$f'(1) = \frac{-1}{4(4)} = \boxed{-\frac{1}{16}}$$