

Question 2

Let

$$f(x) = \sqrt{4x + 1}$$

Use the definition of the derivative to compute $f'(2)$. Show all steps clearly without using shortcut rules.

Question 2 - Solution

We are given:

$$f(x) = \sqrt{4x + 1}$$

Using the limit definition of the derivative:

$$f'(2) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$$

Compute:

$$f(2+h) = \sqrt{4(2+h) + 1} = \sqrt{8+4h+1} = \sqrt{4h+9}$$

$$f(2) = \sqrt{4(2) + 1} = \sqrt{9} = 3$$

So,

$$f'(2) = \lim_{h \rightarrow 0} \frac{\sqrt{4h+9} - 3}{h}$$

To evaluate this limit, multiply numerator and denominator by the conjugate of the numerator:

$$f'(2) = \lim_{h \rightarrow 0} \frac{\sqrt{4h+9} - 3}{h} \cdot \frac{\sqrt{4h+9} + 3}{\sqrt{4h+9} + 3}$$

Apply difference of squares in the numerator:

$$= \lim_{h \rightarrow 0} \frac{(4h+9) - 9}{h(\sqrt{4h+9} + 3)} = \lim_{h \rightarrow 0} \frac{4h}{h(\sqrt{4h+9} + 3)}$$

Cancel h in numerator and denominator:

$$= \lim_{h \rightarrow 0} \frac{4}{\sqrt{4h+9} + 3}$$

Now evaluate the limit:

$$f'(2) = \frac{4}{\sqrt{9+3}} = \frac{4}{3+3} = \boxed{\frac{2}{3}}$$