

Question 1

Let

$$f(x) = \frac{1}{x+2}$$

Use the definition of the derivative to compute $f'(1)$. Show all steps.

Question 1 - Solution

We are given:

$$f(x) = \frac{1}{x+2}$$

Using the definition of the derivative:

$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$$

Compute:

$$f(1+h) = \frac{1}{(1+h)+2} = \frac{1}{h+3}, \quad f(1) = \frac{1}{1+2} = \frac{1}{3}$$

So:

$$f'(1) = \lim_{h \rightarrow 0} \frac{\frac{1}{h+3} - \frac{1}{3}}{h}$$

Find a common denominator for the numerator:

$$\frac{1}{h+3} - \frac{1}{3} = \frac{3 - (h+3)}{3(h+3)} = \frac{-h}{3(h+3)}$$

Now plug back into the limit:

$$f'(1) = \lim_{h \rightarrow 0} \frac{-h}{3(h+3)} \cdot \frac{1}{h} = \lim_{h \rightarrow 0} \frac{-1}{3(h+3)}$$

Now evaluate the limit:

$$f'(1) = \frac{-1}{3(0+3)} = \boxed{-\frac{1}{9}}$$