

Question 8

Let

$$f(x) = \begin{cases} \frac{\sin(3x)}{x}, & x \neq 0 \\ c, & x = 0 \end{cases}$$

Find the value of c that makes $f(x)$ continuous at $x = 0$.

Question 8 - Solution

To ensure continuity at $x = 0$, we require:

$$\lim_{x \rightarrow 0} f(x) = f(0) = c$$

Compute the limit:

$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{x} = \lim_{x \rightarrow 0} \frac{3 \sin(3x)}{3x} = 3 \cdot \lim_{x \rightarrow 0} \frac{\sin(3x)}{3x} = 3 \cdot 1 = 3$$

Thus, to make f continuous at $x = 0$, we must set:

$$\boxed{c = 3}$$