

Question 7

Let the function $f(x)$ be defined by

$$f(x) = \begin{cases} x^2 - 4, & x < 2 \\ kx + 1, & x \geq 2 \end{cases}$$

Determine the value of k that makes $f(x)$ continuous at $x = 2$.

Question 7 - Solution

We are given a piecewise function:

$$f(x) = \begin{cases} x^2 - 4, & x < 2 \\ kx + 1, & x \geq 2 \end{cases}$$

To ensure continuity at $x = 2$, we require:

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$$

Evaluate the left-hand limit:

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (x^2 - 4) = 4 - 4 = 0$$

Evaluate the right-hand limit:

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (kx + 1) = 2k + 1$$

Set the two limits equal for continuity:

$$2k + 1 = 0 \quad \Rightarrow \quad k = -\frac{1}{2}$$

Final Answer:

$$\boxed{k = -\frac{1}{2}}$$