

Question 6

Let the function $f(x)$ be defined as:

$$f(x) = \begin{cases} \frac{\sin(ax)}{x}, & x \neq 0 \\ b, & x = 0 \end{cases}$$

Find the values of constants a and b such that $f(x)$ is continuous at $x = 0$.

Question 6 - Solution

To ensure continuity at $x = 0$, we must have:

$$\lim_{x \rightarrow 0} f(x) = f(0) = b$$

We are given:

$$f(x) = \frac{\sin(ax)}{x}, \quad x \neq 0$$
$$f(0) = b$$

We evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{\sin(ax)}{x} = \lim_{x \rightarrow 0} a \cdot \frac{\sin(ax)}{ax} = a \cdot \lim_{u \rightarrow 0} \frac{\sin u}{u} = a \cdot 1 = a$$

So for continuity, we must have:

$$\lim_{x \rightarrow 0} f(x) = a = b$$

Final Answer:

Any $a \in \mathbb{R}$ and $b = a$
